**Homework 4. Solutions**

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**Ex. 1.** A child is pushing a merry-go-round. The angle through which the merry-go-round has turned varies with time according to where and . (a) Calculate the angular velocity of the merry-go round as a function of time. (b) What is the initial value (i.e. at ) of the angular velocity? (c) Calculate the instantaneous value of the angular velocity a and the average angular velocity for the time interval from to . Show that is not equal to the average of the instantaneous angular velocities at and .

**Solution:**

(a)

(b)

The initial angular velocity is :

At , the angular velocity is:

Average angular velocity:

At time , rad.

At time , .

This value is different to the average of instantaneous angular velocities at and :

**Ex. 2.**

An electric fan is turned off, and its angular velocity decreases uniformly (i.e. motion at constant angular acceleration) from at time to in 4.00 s. (a) Give the initial angular velocity (i.e. at time ) and the angular velocity at time in unit. Then find the angular acceleration in and the number of revolutions made by the motor in the 4.00-s interval. (b) How many more seconds are required for the fan to come to rest if the angular acceleration remains constant at the value calculated in part (a)?

**About “rev” unit.** One revolution (symbol: rev) corresponds to one turn around the axis of rotation, i.e. an angle of . For instance rotate an angle of rad equals . Rotate an angle of equals to 2 rev. **You don’t need** to do convert the angular acceleration in to solve this problem.

**Solution:**

(a) Initial angular velocity :

Angular velocity at time

The angular acceleration is constant :

We obtain the angular acceleration:

We want to describe the change of angular position.

(b) At the time the fan is at rest (” at rest “ means “the angular velocity is zero”).

Another 2.67 s (i.e. ) are required for the fan to come to rest.

**Comment:** the origin of time is arbitrary, you could say that when the fan rotates at 3.333 is the new initial time and apply:

**Ex. 3. Angular velocity for both rotational motions of Earth (its axial spin and its motion around the Sun) are assumed to be constant. The radius of the Earth assumed to be spherical is The orbit (assumed to be circular) radius of the Earth is and is completed in one year, i.e. in . The Earth spins on it during one day, i.e. 86400 s.** Using these astronomical data, calculate (a) the earth’s orbital angular speed (in ) due to its motion around the sun, (b) its angular speed (in )due to its axial spin, (c) the tangential speed (in ) of the earth around the sun (assuming a circular orbit), (d) the tangential speed (in of a point on the earth’s equator due to the planet’s axial spin, and (e) the radial and tangential acceleration components of the point in part (d), with unit: .

**Solution.**

(a) One revolution (i.e. one turn) corresponds to an angle of . The angular velocity is:

(b)

(c)

(d)

(e)

Radial acceleration (i.e. the normal component of the acceleration vector):

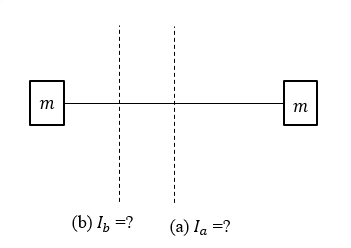
Tangential acceleration (i.e. the tangential component of the acceleration vector):

The motion is assumed to be circular uniform ( is constant), so there is no tangential acceleration, the acceleration vector is directed toward the center of the circular motion.

**Comment about the symbols**: it is not wrong to choose same symbol for both rotational motions, the questions about them are separated, but choosing two different symbols permit to avoid confusion and mistakes.

**Ex. 4.** Small blocks, each with mass m, are clamped at the ends and at the center of a rod of length L and negligible mass. Describe in respect to and the moment of inertia of the system about an axis perpendicular to the rod and passing through (a) the center of the rod and (b) a point one-fourth of the length from one end.

Each block must be treated as point mass body, i.e. a particle.



**Solution.**

(a) Each block is seen as a particle of mass at distance from the axis.

(b)

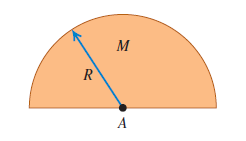
One block is at distance and one block is at distance from the axis:

**Comment:** You can see that we can apply the parallel axis theorem for a system of particles. Here is the first axis pass through the center of mass of the system of the two blocks. The second axis is parallel to the first one. By applying the parallel axis theorem:

where is the mass of the system, and is the distance between the parallel axes.

**Ex. 5.** The moment of inertia about its axis of a disk of any thickness and with uniform mass density is the same that the moment of inertia of a cylinder such as described in classroom (because disk and cylinder have the same shape). A uniform disk of radius R is cut in half so that the remaining half has mass M. (a) What is the moment of inertia of this half about an axis perpendicular to its plane through point A shown in the figure? (b) Why did your answer in part (a) come out the same as if this were a complete disk of mass M?

**You don’t need to demonstrate the moment of inertia of a disk (you can use the result we have found in classroom).**

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**Solution.**

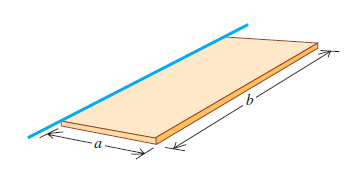
(a)The total moment of inertia for a disk about its axis of rotational symmetry, of mass and radius is:

Each half of the disk has the same moment of inertia about the axis passing through the center of the disk and contribute for the same part to the total moment of inertia. Thus, the moment of inertia for an half-disk about the axis passing through A is:

(b) The reason is that the same mass M is distributed the same way as a function of distance from the axis, for both disks.

**Comment:** You could have calculate the moment of inertia for an half-disk in a way similar to what we have done for a cylinder, but the way here is much more simple.

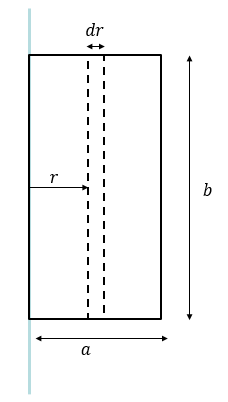
**Ex.6. (a)** Describe the moment of inertia about an axis along edge of a thin rectangular plate of mass , thickness and sides () as shown on the figure in respect to and . The mass density of the plate is uniform. (b) Same question for an axis parallel to the previous axis and passing through then center of mass of the plate.



**Help:** You can describe for infinitesimal volume a thin rectangular part of the plate of volume at distance from the axis.

**Solution:**

(a) We choose for infinitesimal volume a thin rectangular plate of volume at distance from the axis:



The moment of inertia through the axis at the edge of the plate is:

The mass density of the plate, of volume , is :

We obtain:

(b)

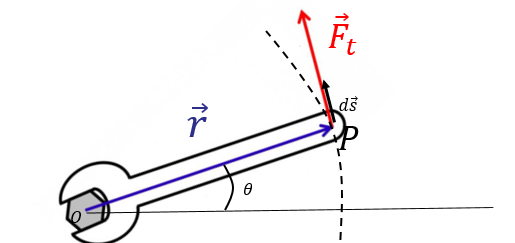
Using the parallel axis theorem,

where is the moment of inertia about the parallel axis passing through the center of mass. Because the plate is thin , we can do the approximation that the distance between both axis is .

We obtain:

**Ex. 7. Rotational work**

As shown on the figure, a tangential force named is applied to a wrenchturning a nut, exerted at point P. We want to describe the work done during the rotational motion of the point P described on the figure from angular position to . Only this tangential force exerts a torque on the wrenchabout point O (friction is ignored). (a) Describe the torque about point O done on the wrench(direction and magnitude in respect to ) (b) Describe the infinitesimal work done by the tangential force during the angular displacement of point P, in respect with . (c) Describe the total work done by the tangential force during the displacement of P from to in respect to the torque exerted by the tangential force about O. (d) The torque by the tangential force about O is also the net torque about O exerted on the wrench(because there are no other forces which exert a torque about O). The net torque about a point of a rotational axis is during a rotational motion is described by where is the moment of inertia about the axis of rotation and is the acceleration vector describing the rotational motion. Please to use this statement to describe the rotational work done by the tangential force from to in terms of , and the angular velocities of P at and .



**Solutions.**

1. The torque about O done by the tangential force is: Using the right-hand rule, the torque is directed perpendicularly to the plane of the paper:

The magnitude of the torque is

(b)

Infinitesimal work done by the tangential force during the infinitesimal displacement of point P is:

(c)The work during the angular displacement from and done by the tangential force is:

We obtain, using is ,

Take care that you cannot put the torque off the integral. The distance is constant during the circular motion of P but the magnitude of the tangential force could change with the time, thus the corresponding torque about O could change during the displacement of P. If is constant, then the torque about O is constant and

(d)

The net torque which is here also the torque done by the tangential force is:

We obtain:

And the angular acceleration is the change rate of the angular velocity :

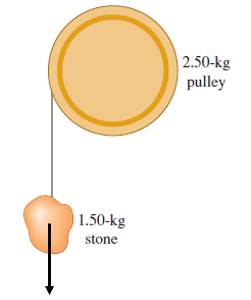
The moment of inertial of the wrench about the axis of rotation is constant during the angular displacement, so we can put it off the integral.

**Comment:** Here is another similarity with translational motion, where the net work on a body in translation is the change of its translational kinetic energy.

If instead of a tangential force, we have arbitrary forces exerted on the rotating wrench, we should obtain the same result because only the tangential component of these forces participate to the rotational motion of the wrench.

**Ex. 8.** A frictionless pulley has the shape of a uniform solid disk of mass 2.50 kg and radius 20.0 cm. A 1.50-kg stone is attached to a very light wire that is wrapped around the rim of the pulley, and the system is released from rest. How far must the stone fall so that the pulley has 4.50 J of kinetic energy?

You don’t have to demonstrate the moment of inertia of the pulley about its axis of rotation, you just have to use the result we have obtained in classroom for a cylinder-like rigid body (cylinder and disk have the same shape).



**Solution:**

To describe how far must the stone fall so that the pulley has 4.50 J of kinetic energy, it is necessary to use the principle of conservation of mechanical energy (friction is ignored, only forces involved are conservative), so it is necessary to describe the kinetic energy of the pulley and the stone.

The rotational kinetic energy of the pulley is :

where is the moment of inertia of the pulley about its axis of rotation and is its angular velocity. The pulley is assumed to have a disk-shape (it has mass and radius ), so its moment of inertia about the axis of rotation is:

The pulley don’t have a translational motion, so its kinetic energy is only rotational kinetic energy. When the rotational kinetic energy of the pulley is 4.50 J, the angular velocity is:

The corresponding velocity (i.e. magnitude of the velocity vector) for the stone (or for a point of the pulley at distance R from the axis of rotation) is:

At this velocity, the stone of mass has kinetic energy which is only translational kinetic energy (the stone don’t have rotational motion):

All the force involved are conservative (the friction is ignored), which means we can use the principle of conservation of mechanical energy to the system “pulley + stone”. The gravitational potential energy of the stone is chosen to be zero when the stone is released: . The gravitational potential energy of the pulley don’t change during its rotational motion (its center of mass is at rest). The rotational kinetic energy of the pulley is zero just after the stone is released.

where is the potential energy of the system “pulley +stone” just after the stone is released, is the total kinetic energy of the system “pulley +stone” just after the stone is released, is the potential energy of the system “pulley + stone” when the rotational kinetic energy of the pulley is 4.50 J, is the kinetic energy of the system “pulley+stone” ”when the rotational kinetic energy of the pulley is 4.50 J. We obtain: